

cant figures, of the integrals of the product of three and four normalized associated Legendre functions. The integral of the product of three functions,  $\Theta(l_1, m_1) \cdot \Theta(l_2, m_2) \Theta(l_3, m_3)$ , is computed for all values of the  $l$ 's from 0 to 8, and all permitted values of the  $m$ 's satisfying the condition  $m_1 + m_2 + m_3 = 0$ , which is the condition for the nonvanishing of the integral of the product of three spherical harmonics. Similarly, the integrals of the product of four functions are tabulated for all values of the  $l$ 's from 0 to 4 and for  $m$ 's satisfying the condition  $m_1 + m_2 + m_3 + m_4 = 0$ .

The computations were carried out on a large-scale digital computer. It is stated that the Gaunt formula, which gives the integral of the threefold product in closed form, was not suitable for programming. Rather, the integrals were obtained in the following straightforward manner: The Legendre polynomials  $P_l(x)$  are generated by means of the recursion formula. The associated polynomials  $P_l^m(x)$  are obtained by  $m$ -fold differentiation. Then three or four of these polynomials,  $(1 - x^2)$  to the appropriate power, and the normalizing factors are multiplied to obtain the integrand; the indefinite integral is taken and the limits  $x = -1$  and  $+1$  are substituted.

To the reviewer's knowledge, these are the first extensive tables of their kind. Previous computations involving the integral of three associated Legendre functions have tabulated either the Clebsch-Gordan coefficients or the  $3j$ -symbols, which are these integrals multiplied by certain other factors.

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**99[L, M].**—HENRY E. FETTIS & JAMES C. CASLIN, *Elliptic Integral of the First Kind and Elliptic Integral of the Second Kind*, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, February 1966. Two mss., each of 36 computer sheets, deposited in the UMT File.

These companion tables consist of 10D values (without differences) of the elliptic integrals of the first and second kinds, respectively, in Legendre's form, namely,  $F(\theta, k)$  and  $E(\theta, k)$ , for  $\theta = 0^\circ(1^\circ)90^\circ$  and  $\arcsin k = 0^\circ(1^\circ)90^\circ$ .

The authors have informed this reviewer that essentially the same subroutine was used in computing these tables on an IBM 1620 with 16-digit arithmetic as was employed in the computation of their published tables [1] of elliptic integrals, wherein the modulus serves as one argument, rather than the modular angle as in the present tables.

It is interesting to note that these new tables in range and precision resemble rather closely the celebrated 9–10D tables of Legendre [2], [3], which may now be considered superseded after more than a century.

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1. HENRY E. FETTIS & JAMES C. CASLIN, *Tables of Elliptic Integrals of the First, Second and Third Kind*, Report ARL 64-232, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, December 1964. (See *Math. Comp.*, v. 19, 1965, p. 509, RMT 81.)

2. A. M. LEGENDRE, *Exercices de Calcul Intégral*, v. 3, Paris, 1816.

3. A. M. LEGENDRE, *Traité des Fonctions Elliptiques et des Intégrales Eulériennes*, v. 2, Paris, 1826. A facsimile reproduction of Tables II and VIII therein appears in K. PEARSON, *Tables of the Complete and Incomplete Elliptic Integrals*, reissued from Tome II of Legendre's *Traité des Fonctions Elliptiques*, London, 1934. (See also ALAN FLETCHER, "Guide to tables of elliptic functions," *MTAC*, v. 3, 1948, pp. 229–281.)